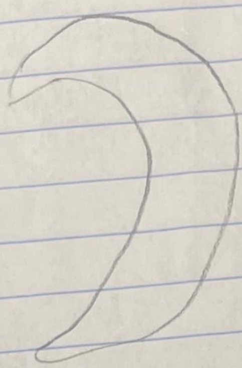
Taemour
Hasan

1

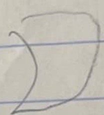
Honors project
Area of a Lune

A lune is a planar region bounded by the arcs of 2 circles



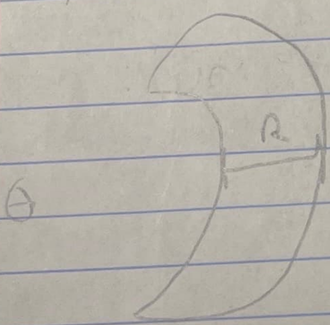
lune

Find formula for the area of a lune in terms of the minimal possible number of variables



Area of big sector: $\frac{1}{2} \cdot R^2 \cdot \theta$

Area of smaller sector: $\frac{1}{2} r^2 \cdot \theta$



big sector

Area of circle $= \pi r^2$

So sector of circle you need θ and some radius of sectors would be bigger so R and for small r, it would be r.

small sector

So to find the area, you just subtract big sector from small sector

Area of Lune $= \left(\frac{1}{2} R^2 \cdot \theta\right) - \left(\frac{1}{2} r^2 \cdot \theta\right)$

Taemoor Hasan

Honors Project 2

Controlled-Source Seismology

1

from S to PD

$$t = \frac{D}{V_1}$$

$$t = \frac{D}{V}$$

so D is distance

$$\text{time} = \frac{\text{Distance}}{\text{velocity}}$$

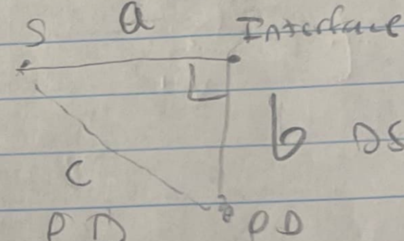
velocity is V_1 ← first layer

time for sound to reach PD from S

2

$$t = \frac{\sqrt{D^2 + 4d^2}}{V_1}$$

sound reflected off the interface - SD



We use pythagoren theorum,

$$a^2 + b^2 = c^2$$

$$D^2 + (2d)^2 = c^2$$

$$D^2 + 4d^2 = c^2$$

$$\text{time} = \frac{\text{Distance}}{\text{velocity}}$$

$c = \sqrt{D^2 + 4d^2}$ ← Distance

V1 is velocity

$$t = \frac{\sqrt{D^2 + 4d^2}}{V_1}$$

3

sound critically refracted along interface to be heard at Pd

$$t = \frac{1}{V_2} D + \frac{2d \cos a}{V_1}$$

$t = \frac{1}{V_2} D + \frac{2d\cos\alpha}{V_1}$ $\quad \frac{1}{V} \quad \frac{2D}{V^2}$ Sound traveling horizontally along interface.

V_2 is horizontal travel velocity

2d is distance going upward and downward ~~that soundwave travels~~

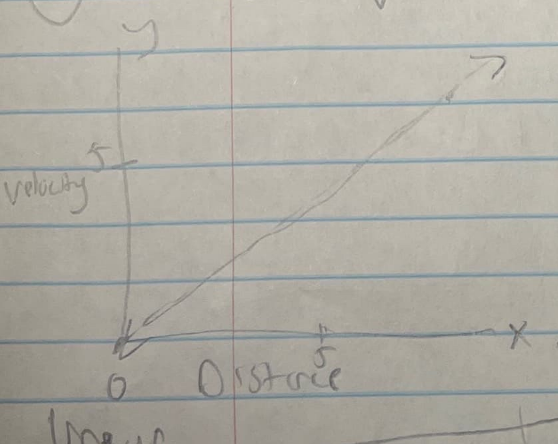
$\cos\alpha$, ← α = angle of refraction when ~~sound goes layer 1 from layer 2,~~

V_1 is velocity found earlier.

to find time = $2d\cos\alpha$ is distance which is altered by $\cos\alpha$, and divided by V_1

So $t = \frac{1}{V_2} D + \frac{2d\cos\alpha}{V_1}$

(4) $t = \frac{D}{V_1}$



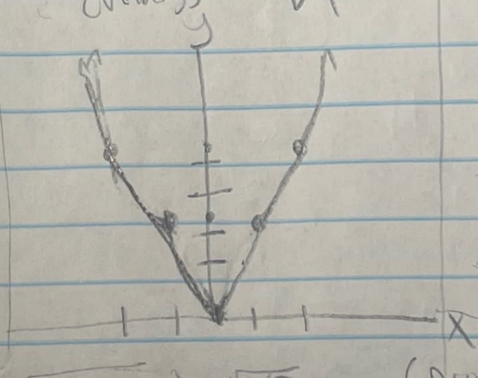
linear

$D = V_1$

$1 = 1$

$2 = 2$

$t = \frac{\sqrt{D^2 + 4d^2}}{V_1}$

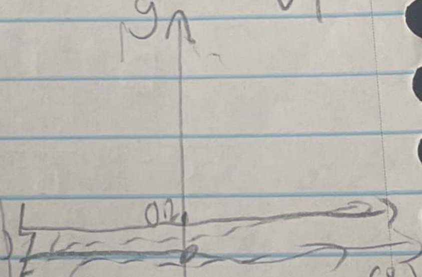


$\sqrt{1^2 + 4(1^2)} = \sqrt{5}$

$\sqrt{2^2 + 4(2^2)} = \sqrt{20} = 2\sqrt{5}$

$4 + 16$

$t = \left(\frac{1}{V_2}\right) D + \frac{2d\cos\alpha}{V_1}$



lets say

$V_1 = 10$ $\quad d = 2$

$V_2 = 20$

$2d(\cos 45) \approx 0.525$

if α is 45

$t = \frac{1}{10^2}(0) + \frac{2 \cdot 2 \cdot 0.525}{10}$

$t = 29.21$

Subject to vary as Inde angle 45°.

Taenur
Hasan

Honors Project 2
Controlled - Source Seismology
continued.

(5) Looking at graphs we can see

V1 is how fast sound travel in top layer

d the width distance of top layer

V2 something as V1 but in lower layer

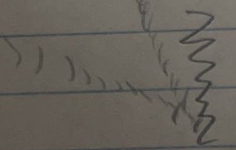
θ sound bends at interface

(6) Real life conditions that can have deviations from ideal are

- sound may be absorbed by different densities of layers

- layers do not have to a straight line. They could be crooked which can affect how sound travel within layers

- Interfaces could be heavily with bumps instead of smooth so the sound could be reflected somewhere else.



Prof Rodriguez Perez

Honors Project 4

Taemoor Hasan

Mathematical Induction

(17) $\left|\sum_{i=1}^{n} a_i\right| \le \sum_{i=1}^{n} |a_i|$ $n \ge 2$ $\sum_{1}^{u+1}$ $|a_i| = \sum_{1}^{u} a_i + (a(u+1))$

Basic ($n=2$)

$\sum_{1}^{2} = |a_i|^2 \; a_i + a_2$ ✓

less than or equal to it is proven

✓ $n \ge 2$

(1) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ for $n \ge 1$

Bass: $P(1)$

$1^2 = \frac{1(2)(3)}{6} = \frac{6}{1}$ ✓

$(u) \rightarrow (u+1)$

induction

$1^2 + 2^2 + \dots u^2 + (u+1)^2$

$= u \frac{(u)(u+1)(2u+1)}{6} + (u+1)^2$

$\frac{(u+1)(u+2)(2u+3)}{6} = \frac{(u+1)(u+2)(2u+3)}{6} = \frac{u(u+1)(2u+1) + 6(u+1)^2}{6}$ ✓

(2) $\prod_{i=2}^{n} \left(1 - \frac{1}{i}\right) = \frac{1}{n}$ $n \ge 3$

Basis ($n=3$)

$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) = \frac{1}{3}$

$\frac{1}{2} \times \frac{2}{3}$

$\frac{1}{3}$ ✓

$(u) \rightarrow (u+1)$

Inductive:

$\prod_{i=2}^{u+1} \left(1 - \frac{1}{i}\right) = \left(1 - \frac{1}{i}\right) \times \left(1 - \frac{1}{u+1}\right)$

basis

$\frac{1}{\cancel{u}} \times \frac{u}{u+1}$

$\frac{1}{u+1} = \frac{1}{u+1}$ ✓

(3) $2^n > n$ for $n \ge 0$

Bass $P(0)$

$2^0 > 0$

$1 > 0$

✓

Inductive $P(u) \rightarrow P(u+1)$

$2^{u+1} = 2 \times 2^u > 2^u$

$u + u = 2u$

$2u > u$ $u > 1$ ✓

(9.) $\sum_{i=0}^{n} 2^i = 2^{n+1}$ for $n \ge 1$

Base case not true

Base $P(n=1)$

Invalid

$2^1 + 2^0 = 2^2$

$2 + 1$ X no

(4) $n! \geq 2^{n-1}$ $n \geq 1$

Base $(n=1)$

$1! \geq 2^0$ ✓

Induction $P(k) \to P(k+1)$

$(k+1)! = (k+1) \times k! \geq (k+1) \cdot 2^{k-1}$

$2 \times 2^{k-1}$

2^k ✓

(5) $2^n < n!$ for $n \geq 4$

Base $(n=4)$

$2^4 < 4!$

24 ✓

induction $P(k) \to P(k+1)$

$2^{k+1} = 2 \cdot 2^k < 2 \cdot k!$

$(k+1) \cdot k!$

$(k+1)!$ ✓

(6) $a < b$ $n \geq 1$

then $a^n < b^n$

Base $(n=1)$ ✓

$a^1 < b^1$

induction $P(k) \to P(k+1)$

$a^{k+1} = a^k a < b^k \cdot b =$

b^{k+1} ✓

(7) $2^n > n^2$ $n \geq 5$

Base $(n=5)$

$2^5 > 25$ ✓

Induction $P(k) \to P(k+1)$

$2^{k+1} = 2 \cdot 2^k > 2 \cdot k^2$

$(k+1)^2$ ✓

$2k^2 >$

$2k+1 >$

$(k+1)^2$ for

$k \geq 5$

(8) $n^3 < n!$ for $n \geq 6$

Base $n=6$

$6^3 < 6!$

$216 < 720$ ✓

(Induction)

$P(k) \to P(k+1)$

$(k+1)^3 = k^3 + 3k^2 + 3k + 1 < k! +$

$3k^2 + 3k + 1$

$k! + 3k^2 + 3k + 1 < (k+1)!$ ✓

(10) $\sum_{i=1}^{n} \frac{i}{(i+1)!} = 1 - \frac{1}{(n+1)!}$ for $n \geq 1$

Base $(n=1)$

$\frac{1}{(1+1)!} = \frac{1}{2}$ ✓

$1 - \frac{1}{(1+1)!} = \frac{1}{2}$ ✓

induction $P(k) \to P(k+1)$

$\sum$ $i=1$ to $k+1$ $i/(i+1)! = \sum(i=1$ to $k)$ $i/(i+1)! + (k+1)/(k+1+1)!$

$1 - \frac{1}{(k+1)!} + \frac{(k+1)}{(k+2)!} \to \frac{k+2-1}{(k+2)!} + \frac{(k+1)}{(k+2)!}$

$\frac{2k+2}{(k+2)!}$ $\frac{2(k+1)!}{(k+2)!} \times (k+2)$ $\frac{2(k+1)}{(k+2)!\,(k+2)}$

✓ (1) $\frac{2(k+1)!}{(k+2)!} = \frac{(k+2)!}{(k+2)!}$

Hasan

Honors Project 10: The Human Cough

(1)

$$F = \int_0^R 2\pi r v \, dr$$ ← integral of velocity function $v(r)$

$$F = \frac{\pi P R^4}{8 u L}$$ → Poiseuille's law

$$\int_0^R 2\pi \left[P(2uL)(R^2 - r^2) \right] dr$$

$$F = 2\pi P(2uL) \int_0^R r(R^2 - r^2)\, dr$$

$$F = 2\pi P(2uL)\left[\frac{1}{3}R^3 - \frac{1}{5}R^5\right]$$

$$F = \frac{2}{3}\pi P(2uL)\left(R^3 - \frac{R^5}{5}\right)$$

$$F = \frac{2}{3}\pi P(2u)R^3\left(1 - \frac{R^2}{5}\right)$$

$$F = \frac{2}{3}\pi P(2uL)R^3 \frac{(5 - R^2)}{5}$$

$$F = \frac{10}{3}\pi P(2uL)R^3\left(1 - \frac{R^2}{5}\right)$$

$$F = \frac{10}{3}\pi P(2uL)(R^4 - R^2)$$

$$F = \frac{10}{3}\pi P(2uL)R^4$$

$$\frac{10}{3}\pi (P 2uL)R^2$$

Avg Flow

$$\frac{F}{\pi}R^2 = \left[\left(\frac{10}{3}\pi P(2uL)R^4 - \frac{\frac{10}{3}\pi P(2uL)R^2}{\pi R^2}\right)\right.$$

$$\frac{F}{\pi}R^2 = \frac{10}{3}P(2uL)R^2 - \frac{10}{3}P(2uL)$$

$$\frac{F}{\pi}R^2 = \frac{10}{3}P(2uL)\left[R^2 - 1\right]$$

Proportional to $R^2 - 1$

(2)

$$\frac{F}{\pi}R^2 = \left(\frac{10}{3}\right)(2uL)(R^2 - 1)$$

$$\frac{d}{dr}\left(\frac{10}{3}\right)(2uL)\left(R^2 - 1\right] = 0$$

$$\left(\frac{10}{3}\right)(2uL)(2R) = 0$$

$$2R = 0$$

minimum → $R = 0$

CP

$$\left(\frac{10}{3}\right)(2uL)(2R) =$$

$$2R = 0$$

$$R = 0$$

Predicting chg.

0%

radius no ch

So difference btwn

33%

Taemour Honors Proj
Hasyn IS: Oprom of Disel Engine

(2g) $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}=\frac{P_3V_3}{T_3}=\frac{P_4V_4}{T_4}$ IS gas lw $PV=nRT$

use effcy formla ad plug $E=1-\frac{1}{r}\left(\frac{T_1}{T_2}\right)^{\gamma-1}\cdot\left(\frac{T_3}{T_4}\right)^{\gamma-1}$

(1) (A:) $P=\frac{P_1V_1^{\gamma}}{V^{\gamma}}$ $P=P_1\left(\frac{V_1}{V}\right)^{\gamma}$

$PV^{\gamma}=kc$ $P_1V_1^{\gamma}=kc$. Solvs for P_1

// $P=P_1\left(\frac{V_1}{V}\right)^{\gamma}$

(B) $P=\frac{P_1V_2^{\gamma}}{V^{\gamma}}$ $PV^{\gamma}=kp$ $V=V_2$

// $P=P_1\left(\frac{V_2}{V}\right)^{\gamma}$

(C) $\left(\frac{V_1}{V_4}\right)^{\gamma}=\frac{P_4}{P_1}$ and $\left(\frac{V_2}{V_3}\right)^{\gamma}=\frac{P_3}{P_2}$

$PV^{\gamma}=kc$

- $(V_4,P_4)=P_4V_4^{\gamma}=kc$
- $(V_1,P_1)=P_1V_1^{\gamma}=kc$ $\longrightarrow$ $\frac{V_1^{\gamma}}{V_4^{\gamma}}=\frac{P_4}{P_1}$ //
- $(V_2,P_2)=P_2V_2^{\gamma}=k\phi$ $\longrightarrow$ $\frac{V_2^{\gamma}}{V_3^{\gamma}}=\frac{P_3}{P_2}$ //
- $(V_3,P_3)=P_3V_3^{\gamma}=k\phi$

(2) (A:) $W_c=-\int_{l=0}^{L}F\,dl=-\int_{l=0}^{L}PA\,dl=-\int_{V=V_4}^{V_1}P\,dV$

work dury comp strok V_4 to V_1,

work done as integral of presure

W_c is work done dury comp stroke from V_4 to V_1.
$-\int_0^L PA\,dl$ represents work done as intergral of presure times the piston area over the displacment; A constant so dl is $A\,dV$.
$W_c=-\int_0^L PA\,dl=-\int_{V_4}^{V_1}P\,dV$

(2B) $W_c = \frac{1}{r-1}(P_1V_1 - P_4V_3)$

Comp curve

$P = P_1\left(\frac{V_1}{V}\right)^r$ sub P.

$W_c = -\int_{V_4}^{V_1} P\,dV = -\int_{V_4}^{V_1} P_1\left(\frac{V_1}{V}\right)^r dV$

$W_c = -P_1V_1\int_{V_4}^{V_1} \frac{1}{V^r}\,dV$

$\frac{V_1^r}{V_4^r} = \frac{P_4}{P_1}$

$W_c = \frac{P_1V_1}{r-1}\left(\frac{1}{V_1^{r-1}} - \frac{1}{V_4^{r-1}}\right)$

$W_c = \frac{1}{r-1}(P_1V_1 - P_4V_3)$ //

(2C) $W_\varphi = P_1(V_2 - V_1) + \int_{V=V_2}^{V_3} P\,dV$

Pow curve

$P = P_1\left(\frac{V_2}{V}\right)^r$ sub P

$W_\varphi = \int_{V_1}^{V_3} P\,dV = \int_{V_1}^{V_2} P_1\left(\frac{V_2}{V}\right)^r dV$

$W_\varphi = P_1V_1\int_{V_1}^{V_3} \frac{V_2^r}{V^{r+1}}\,dV$

$W_p = \frac{P_1}{r-1}(V_2 - V_1 + V_3 - V_2)$

$W_p = \frac{1}{r-1}\left(P_1(V_2 - V_1) + P_1(V_3 - V_2)\right)$ //

2d) $\frac{V_2^r}{V_3^r} = \frac{P_3}{P_2}$

$V_2 = \left(\frac{P_3}{P_2}\right)^{1/r} V_3$

$W_\varphi = \frac{1}{r-1}(rP_1(V_2 - V_1) + P_1V_1 - P_3V_3)$ //

$W_p = \frac{1}{r-1}\left(rP_1\left(\left(\frac{P_3}{P_2}\right)^{1/r} - 1\right)V_3 + P_1V_1 - P_3V_3\right)$

2e) ~~$W_p - W_c$~~

$W_p - W_c = \frac{1}{r-1}(rP_1(V_2 - V_1) + P_1V_1 - P_3V_3) - \frac{1}{r-1}(P_1V_1 - P_4V_3)$

$W_\varphi - W_c = \frac{1}{r-1}(rP_1(V_2 - V_1) + P_1V_1 - P_3V_3 - P_1V_1 + P_4V_3)$ //

2f) $E = \frac{W_p - W_c}{W_p} = 1 - \frac{W_c}{W_p}$

$E = 1 - \frac{W_c}{W_p}$

$E = 1 - \frac{\frac{1}{r-1}(P_1V_1 - P_4V_3)}{\frac{1}{r-1}(rP_1(V_2 - V_1) + P_1V_1 - P_3V_3)}$ //

$E = 1 - \frac{P_4V_3}{rP_1(V_2 - V_1) + P_1V_1 - P_3V_3}$